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Robust Computation of Optical Flow under Non-uniform Illumination Variations

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Abstract

In this paper, an energy minimization method is proposed to estimate the optical flow of an image sequence in the presence of non-uniform illumination variations. The energy function is formulated by combining a data constraint energy that considers the illumination variations and a smoothness constraint, which minimizes the pixel-to-pixel variation of the velocity and illumination fields. Minimization of this energy function is equivalent to solving a linear system, which is accomplished by using an incomplete Cholesky preconditioned conjugate gradient algorithm. A dynamic weighting scheme, which considers the statistical properties of estimated optical flow, is also combined with this algorithm to improve the robustness of our algorithm. This algorithm has been successfully applied to synthetic and real image sequences and some experimental results demonstrate that this algorithm can estimate the optical flow under non-uniform illumination variations accurately.

1. Introduction

Motion analysis of image sequences is a fundamental problem in digital video processing and computer vision. Although optical flow is somewhat different from the motion fields [1], optical flow can provide very important information about the estimation of 3D velocity fields, the segmentation of image sequence and the reconstruction of 3D objects. In the early researches, most researchers assumed that the brightness be constant over the image sequences and obtained good estimation of the optical flow for this ideal situation. In recent years, some researchers tried to relax this brightness constancy assumption and developed some algorithms to estimate the optical flow in the presence of illumination variations [2, 3, 4, 5, 6, 7]. Nomura et. al. developed an extended optical flow equation by assuming that the brightness distribution function could be modeled as a non-uniform illumination function multiplying an image brightness function under uniform illumination [2]. Negahdaripour proposed a revised definition of optical flow which is a complete representation of geometric and radiometric variations in dynamic imagery [3]. Haussecker and Fleet used several physical models that described brightness variations to compute the optical flow [4]. Hampson and Pesquet [5] introduced a brightness variation factor in their pel-recursive algorithm to analyze the motion of an image sequence. Nomura [6] and Zhang et. al. [7] used the extended optical-flow equation developed by Nomura et. al. [2] with different computation models to estimate the motion vector fields. However, to simplify the computation, most of them used the assumption of constant parameters in a local neighborhood to estimate the optical flow. This framework could not give an accurate result when the singular case was encountered. Lai and Vemuri proposed a regularization method that considers the motion of all pixels in the image simultaneously [8]. This algorithm could give an accurate and dense estimation of the optical flow. However, they did not consider the condition of varying illumination in their method. Hence, in this paper, we propose an energy minimization method that generalizes Lai and Vemuri’s algorithm to estimate the optical flows of image sequences under non-uniform illumination variations. A robust estimation scheme is also included into this algorithm to increase the robustness of our algorithm. The detailed description of our algorithm is given in the following sections.

2. Energy Minimization Formulation

The traditional optical flow equation is written as follows:

\[ I_x(x, y, t)u(x, y, t) + I_y(x, y, t)v(x, y, t) + I_t(x, y, t) = 0, \]  

(1)
where \( I(x,y,t) \) is the image intensity function, \((u,v)\) is the motion vector and \( I_x, I_y \) and \( I_t \) are the partial derivatives of the image intensity function with respect to \( x, y \) and \( t \). This equation is derived based on the first-order Taylor series approximation and the assumption of brightness constancy. To extend the optical flow estimation problem to account for non-uniform illumination changes, Normura et al. [2] derived a generalized optical flow equation. In this equation, they assumed the image intensity function could be represented as the multiplication of a non-uniform illumination function and an image intensity distribution function under uniform illumination. When the illumination variations were dominated in the temporal domain, the following equation can be obtained.

\[
\begin{align*}
I_i(x,y,t)u(x,y,t) + I_j(x,y,t)v(x,y,t) + I_t(x,y,t) + \beta(x,y,t)I(x,y,t) = 0,
\end{align*}
\]

(2)

where \( \beta(x,y,t) \) is an unknown function describing the illumination variations. Based on this generalized optical flow equation and a smoothness constraint that minimizes the pixel-to-pixel variation of the motion vectors \((u,v)\) and the illumination variation function \( \beta(x,y,t) \), we can formulate the optical flow estimation problem by minimizing an energy function of the following form:

\[
E(u) = \sum (I_i \cdot u_i + I_j \cdot v_j + I_t + \beta I)^2 + \lambda \sum (u_i^2 + u_j^2 + v_i^2 + v_j^2 + \beta_i^2 + \beta_j^2),
\]

(3)

where the subscript \( i \) denotes the \( i \)-th location, subscripts \( x, y \) and \( t \) denote the partial derivatives along the corresponding directions, \( \lambda \) is a constant and the vector \( u \) is the concatenation of all the components \( u_i, v_i \) and \( \beta_i \).

Minimization of this energy function can lead to accurate estimation of the optical flow for an image sequence in the presence of non-uniform illumination variations.

### 3. Incomplete Cholesky Preconditioned Conjugate Gradient Algorithm

The energy function described in the previous section can be rewritten as a quadratic form and minimization of this energy function is equivalent to solving a linear system \( Ku=b \) where the matrix \( K \) is a symmetric positive-definite matrix and it takes the following 3-block structure.

\[
K = \begin{bmatrix}
\lambda K_x + E_{xx} & E_{xy} & E_{xz} \\
E_{yx} & \lambda K_y + E_{yy} & E_{yz} \\
E_{zx} & E_{zy} & \lambda K_z + E_{zz}
\end{bmatrix} \in R^{nm \times nmn}.
\]

(4)

The matrix \( K_x \in R^{m \times m} \) is the discrete 2D Laplacian matrix from the smoothness constraint and \( E_{xx}, E_{xy}, E_{xz}, E_{yy}, E_{yz}, E_{zx} \) and \( E_{zz} \) are all \( mn \) \( \times \) \( nmn \) diagonal matrices with diagonal entries \( I_{xx}, I_{xz}, I_{yy}, I_{yz}, I_{zx}, I_{zz} \) and \( I_t \) respectively. \((m \times n) \) is the image width and height. This linear equation can be solved by a preconditioned conjugate gradient algorithm with an incomplete Cholesky preconditioner \( P \) [8, 9]. The detailed steps of our algorithm are described as follows:

- **Initialize** \( u_0 \) ; compute \( r_0 = b - Ku_0 \); \( k = 0 \).
- **Solve** \( Pz_k = r_k \); \( k = k+1 \).
- **If** \( k = 1 \), \( p_1 = z_0 \); else compute \( \beta^k = r_k^T z_{k-1} \), \( \beta^k = \alpha^k/\beta^k \), and update \( p_k = z_{k-1} + \beta p_{k-1} \).
- **Compute** \( \alpha^k = r_k^T z_{k-1} \), \( \beta^k = p_k^T K p_k \), and \( \alpha^k = \alpha^k/\beta^k \).
- **Update** \( r_k = r_k - \alpha_k p_k \), \( u_k = u_{k-1} + \alpha_k p_k \).
- **If** \( r_k \approx 0 \), stop; else go to step 2.

The preconditioner matrix \( P \) is selected as the incomplete Cholesky factorization of the matrix \( K \). That is \( P = LL^T \approx K \). In this approximation, only the nonzero elements in \( K \) are equal to those of the preconditioner matrix \( P \) at the corresponding locations. The matrix \( L \) has the following form:

\[
L = \begin{bmatrix}
L_{11} & 0 & 0 \\
L_{21} & L_{22} & 0 \\
L_{31} & L_{32} & L_{33}
\end{bmatrix} \in R^{nm \times nmn},
\]

(5)

where the submatrices \( L_{ij} \) are of size \( mn \) \( \times \) \( nmn \) and take the following structure.

\[
L_{ij} = \begin{bmatrix}
G^{(1)}_{ij} \\
H^{(1)}_{ij} G^{(2)}_{ij} \\
\vdots \\
H^{(n-1)}_{ij} G^{(2)}_{ij} \\
H^{(n)}_{ij} G^{(2)}_{ij}
\end{bmatrix},
\]

(6)

for \((i,j) \in \{(1,1),(2,2),(3,3)\}\)

\[
L_{ij} = \begin{bmatrix}
G^{(1)}_{ij}^T \\
H^{(1)}_{ij} G^{(2)}_{ij}^T \\
\vdots \\
H^{(n)}_{ij} G^{(2)}_{ij}^T \\
G^{(2)}_{ij}^T
\end{bmatrix},
\]

(7)

for \((i,j) \in \{(2,1),(3,1),(3,2)\}\). The matrices \( G^{(i)}_{ij} \) and \( H^{(i)}_{ij} \) are

\[
G^{(i)}_{ij} = \begin{bmatrix}
\alpha^{(i)}_{1,1} \\
\alpha^{(i)}_{1,2} \\
\vdots \\
\alpha^{(i)}_{n-1,n} \alpha^{(i)}_{n,n}
\end{bmatrix} \in R^{n \times n},
\]

(8)
where \( \mathbf{W}^{(k)} \) is a diagonal weighting matrix with diagonal elements \( w_{ij}^{(k)} \) denoting the weighting for the \( i \)th data constraint at the \( k \)th iteration. For the Lorentzian function used in this paper, the weighting \( w_{ij}^{(k)} \) is given by
\[
w_{ij}^{(k)} = \frac{2\sigma_{ij}^{(k)}}{2\sigma_{ij}^{(k)} + r_{ij}^{(k)^2}} ,
\]
where \( \sigma_{ij}^{(k)} \) is the standard deviation of the residual \( r_{ij}^{(k)} \) at the \( k \)th iteration. The residual is defined as follows:
\[
r_{ij}^{(k)} = I_{ij}u_{ij}^{(k)} + I_{ij}v_{ij}^{(k)} + I_{ij} + \beta_{ij}^{(k)}I .
\]
Combining the robust estimation scheme with the incomplete Cholesky preconditioned conjugate gradient algorithm, the final algorithm is summarized as follows.

1. \( k = 0 \); Form the matrix \( \mathbf{K}^{(0)} \) and vector \( \mathbf{b}^{(0)} \).
2. Apply incomplete Cholesky preconditioned conjugate gradient algorithm to solve \( \mathbf{K}^{(k)}\mathbf{u}^{(k+1)} = \mathbf{b}^{(k)} \) for \( \mathbf{u}^{(k+1)} \).
3. If \( \mathbf{u}^{(k+1)} \approx \mathbf{u}^{(k)} \) stop; else go to step 4.
4. Calculate \( \mathbf{W}^{(k)} \).
5. \( k = k + 1 \); Form the matrix \( \mathbf{K}^{(k)} \) and vector \( \mathbf{b}^{(k)} \). Go to step 2.

### 5. Experimental Results

To evaluate the performance of our algorithm, we used some synthetic image sequences as our test data. These image sequences were wildly used by many researches [7, 8, 12]. The angular error measure used by Barron et. al. [12] is adopted as our performance measure and is a basis for comparison with other reported results. Before testing, a modification is made to improve the performance of our algorithm. That is, each data constraint is divided by \( \sqrt{I_{ij} + I_{ij}^2 + I_{ij}^2 + c} \) where the constant \( c \) was used to avoid the problem of dividing by zero. This normalization factor can reduce the large weighting at the high gradient and high brightness locations.

In this paper, the performance of Yosemite image sequence is shown. The intensity values in the sky region of Yosemite sequence do not satisfy the brightness constancy assumption and therefore is a good example that can be used to test our algorithm. Figure 1 shows one frame of this image sequence and Figure 2 shows the exact optical flow of this image sequence. Figure 3 is the estimated optical flow of proposed algorithm. Table 1 lists the performance of our algorithm and other reported methods. From this table, we can see that our algorithm can estimate the optical flow accurately with 100% density. Among all the results with optical flow estimation at 100% density, the proposed method has the best accuracy performance.
Table 1. Summary of Yosemite results [7, 8, 12]

<table>
<thead>
<tr>
<th>Technique</th>
<th>Avg. error (deg)</th>
<th>St. dev. (deg)</th>
<th>Density (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horn &amp; Schunck (Modified)</td>
<td>9.78</td>
<td>16.19</td>
<td>100</td>
</tr>
<tr>
<td>Uras et. al.</td>
<td>8.94</td>
<td>15.61</td>
<td>100</td>
</tr>
<tr>
<td>Fleet &amp; Jepson</td>
<td>4.63</td>
<td>13.42</td>
<td>34.1</td>
</tr>
<tr>
<td>Lucas &amp; Kanade</td>
<td>4.28</td>
<td>11.41</td>
<td>35.1</td>
</tr>
<tr>
<td>Weber &amp; Malik</td>
<td>4.31</td>
<td>8.66</td>
<td>64.2</td>
</tr>
<tr>
<td>Lai &amp; Vemuri</td>
<td>7.81</td>
<td>14.57</td>
<td>100</td>
</tr>
<tr>
<td>L. Zhang et. al.</td>
<td>5.59</td>
<td>11.24</td>
<td>100</td>
</tr>
<tr>
<td>Proposed method</td>
<td>4.58</td>
<td>7.93</td>
<td>100</td>
</tr>
</tbody>
</table>

Figure 1. Yosemite image sequence

Figure 2. Exact optical flow of Yosemite

Figure 3. Estimated optical flow of Yosemite

6. Conclusions

In this paper, we proposed an algorithm to estimate the optical flow of image sequences under non-uniform illumination variations. This algorithm is an energy minimization method that used the incomplete Cholesky preconditioned conjugate gradient algorithm for efficient minimization of a new energy function. A robust scheme was included in this framework to improve the robustness of our algorithm. The experimental results demonstrated that our algorithm could estimate the optical flow of image sequence accurately in the presence of non-uniform illumination variations.