The following manuscript was published in

MAT BASED THINNING FOR LINE PATTERNS

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Reducing branching effect and increasing boundary noise immunity are of great importance for thinning patterns. An approach based on medial axis transform (MAT) to obtain a connected 1-pixel wide skeleton with few redundant branches is presented in this paper. Though the obtained skeleton by MAT is isotropic with few redundant branches, however the skeleton points are usually disconnected. In order to rend the merits of the MAT and avoid its disadvantages, the proposed approach is composed of distance-map generation, grouping, ridge-path linking, and refining to obtain the connected 1-pixel wide thin line. The ridge-path linking strategy can guarantee the skeletons connected, whereas the refining process can be readily performed by a conventional thinning process to obtain the 1-pixel wide thinned pattern. The performances investigated by branching effect, signal-to-noise ratio (SNR), and measurement of skeleton deviation (MSD) confirm the feasibility of the proposed MAT-based thinning for line patterns.

Keywords: MAT; medial axis transform; skeletonization; thinning; line pattern.

1. Introduction

Thinning plays a significant role in the fields of image processing and pattern recognition, e.g., optical character recognition, fingerprint recognition, medical image processing, posture recognition, etc. These applications show that reducing patterns to a thin-line representation can get the merits of reducing data amount as well as making shape analysis more easily since it is very convenient to perform the feature-point extraction and line tracking on a thin-line. Thinning is an iterative edge-point erosion technique and usually performed by a set of thinning templates or thinning rules, thus also called rule-based thinning. Based on the isotropic and topological property, the fundamental requirement of a thinning algorithm is to yield a connected and 1-pixel wide thin line. The connectivity can be of 4- or 8-connected depending on the operating procedure. Of course the perfect 8-connectivity is usually desired in the thinning research. For example, like the work by Chen and Hsu, Li and Wang gave a comment on the fast parallel thinning algorithm presented by Zhang and Suen. Many valuable papers dealing

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with the theoretical and experimental aspects of thinning methodologies can be found in the book collected by Suen and Wang. Wang and Zhang proposed a structure-preserving, fast and flexible thinning method (WZ method), which is able to perform the thinning algorithm in serial and parallel. Nagendra prasad et al. derived an improved algorithm to speed up the WZ method. Zhang and Wang presented a generic approach for the analysis and design of parallel thinning algorithms, where 4-subcycle, 2-subcycle, and 1-subcycle thinning algorithms are developed for discussion. In addition, Lam and Suen gave a comprehensive survey of thinning algorithms and examined the effects of different parallel thinning algorithms on an OCR system.

Though so many thinning algorithms and their improved versions have been presented over half century and applied in many fields, some interesting issues such as rotation invariant, branching effect, and boundary noise immunity are still worthy of studying and investigated by numerous researchers in recent decades. Some literatures have noticed and investigated this phenomenon. Based on the comparable performance of pseudo 1-subcycle parallel thinning algorithm, Chen developed a hidden deletable pixel detection algorithm for obtaining the bias-reduced skeletons. Ahmed and Ward developed a rule-based thinning algorithm for the preservation of rotation invariant in character recognition application, and Rockett presented its improved version. However the effectiveness of reducing the branching effect and increasing the boundary noise immunity have been restricted by a rule-based thinning algorithm since the size of thinning templates is very small, usually between $3 \times 3$ and $5 \times 5$. Due to the fact that the rule-based thinning iteratively removes the contour pixels until the 1-pixel wide skeleton is extracted, the removing manipulation is very vulnerable to the variation of contour points. Little noise on the contour points may probably give rise to unwanted branches, and thus disturb the topology of skeleton. Accordingly boundary noise immunity of rule-based thinning algorithm is rather limited.

In order to increase the boundary noise immunity, some branch-pruning methods of post treatment have been proposed. Krinidis and Krinidis proposed a method based on signal processing EMD (Empirical Mode Decomposition) to prune the spurious branches generated by rule-based thinning, which iteratively determines the contour points’ maximum and minimum angles, calculates the mean envelope subtracted from previous envelopes to obtain several intrinsic mode functions until the intrinsic mode function becomes monotonic. Based on the calculated angles from the obtained intrinsic mode functions, the angle map is constructed to determine the important angle set. Finally the spurious branches whose end points do not belong to the corresponding points of the angle set will be pruned. Shen et al. proposed a method to prune the redundant branches of thinning result by means of identifying the two opposite corner points of a protrusion on the contour and locating a ghost point that equally divides the arc between the two corner points. Based on these points, the other point falling on the skeleton and the separated line between the two corner points are used to construct upper and lower triangles.
The so-called bending potential ratio is used to determine whether the branch is redundant or not. These algorithms are developed for determining if the branches shall be pruned, and thus can be regarded as the post pruning treatment.

Different to rule-based thinning that removes contour points to extract the skeleton, the medial axis transform (MAT) which originates from Blum and Nagel’s skeletal concept calculates every foreground point’s shortest distance with respect to the nearest background to construct distance map. By the distance map and comparing with its eight neighbors, the local point with the largest distance becomes the skeleton point. The skeleton points are usually located at the midpoint area of the pattern, which meets the isotropic requirement of skeletonization. Unlike rule-based thinning, this extracted skeleton is not very sensitive to the variation of the boundary and little noise seldom causes spurious branches. Even though the MAT possesses such an advantage, for a digital image these skeleton points are usually discrete, disconnected, as well as not 1-pixel wide. Pursuing a connected 1-pixel wide thin line via MAT is thus the main goal of this study.

In order to obtain the connected skeleton, Shih and Pu proposed a scheme in keeping track of the maximum Euclidean distance values, where the corner points are also involved as the skeleton points. Starting from the skeleton points and observing the local point’s Euclidean distance change with respect to its eight neighbors, the method finds out the directional neighbors, and iteratively conducts tracking the ascending uphill points and descending downhill points until no points are involved. However by this tracking method, many unwanted ridges will be involved, in addition involving the corner points that locate on the contour will yield the unwanted branches.

Yan and Zhou also proposed a skeletonization algorithm which morphologically applies five templates on the distance map, the local point having largest distance value which satisfies one of the templates will be eliminated. The distance map is made by chessboard, and it may generate more spurious points than Euclidean based metric. Inevitably, this method will generate redundant branches. Chang proposed a skeleton extracting method similar to that of Shih and Pu, which also relies on the observation to keep track of the ascending and descending ridges. In this way, the unwanted branches will also be involved unavoidably.

For increasing the boundary noise immunity and obtaining a 1-pixel wide thin line, this paper presents a MAT-based thinning algorithm with Euclidean distance metric for producing the distance map. The Euclidean distance is adopted to reflect a more accurate metric. A ridge-path linking algorithm with a newly defined cost function is developed for systematically determining the best ridges among skeleton points. After obtaining the connected thinned results, the 1-pixel wide thin line can be readily refined by a conventional thinning treatment. The performances of the proposed approach are investigated by branching effect and boundary noise with signal-to-noise ratio as well as measurement of skeleton deviation.

The rest of this paper is organized as follows. In Section 2, followed by the fundamental definitions and investigations of MAT, the proposed approach including
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generation of distance map, grouping, ridge path linking and refining is presented. Section 3 shows experimental results and comparisons. Branching effect and boundary noise immunity are used for analyses and discussions. The conclusions and future works are finally drawn in Section 4.

2. Proposed Method

Before presenting the proposed method, some fundamental definitions about medial axis transformation (MAT) are introduced and discussed first. For a digital pattern consisting of pattern pixels and background pixels, let \( p \) and \( q \) be pattern pixels with coordinates \((x_1, y_1)\) and \((x_2, y_2)\), respectively. In general we may have the \( D_4(p, q) \) distance (city-block distance), \( D_8(p, q) \) distance (chessboard distance), and \( D_E(p, q) \) distance (Euclidean distance) between \( p \) and \( q \) for computation.

According to the definition by Blum\textsuperscript{2}, the shape’s MAT in a digital pattern can be defined as follows. For each pattern pixel \( p \), there exists a nearest neighbor to the boundary of the shape. The distance between the pixel \( p \) and the nearest boundary is denoted as \( d(p) \) or called as \( p \)’s \( d \)-value in this paper. If \( p \) has at least two such neighbors, it belongs to the medial axis (MA) of the shape. In other definition, the MA consists of those pattern pixels whose distances to the nearest boundary are local maxima. The “nearest” may be dependent on the metrics as defined above. Before identifying the MA, a distance map (D-map) containing the metric information \( d(p) \) for each pattern pixel \( p \) should be produced as illustrated in Fig. 1. Here each numbered block denotes a pattern pixel. \( d(p) = 1.0 \) represents \( p \) is one pattern neighbor of the shape’s boundary. Obviously, the distance relationship among these metric is \( D_4(p, q) \geq D_E(p, q) \geq D_8(p, q) \) as investigated by the literatures.\textsuperscript{29,8,13} Considering pixels \( p_0, p_1, \) and \( p_2 \) have the coordinates \((1, 1), (2, 2), \) and \((2, 1)\), respectively, we will have the following results. The metrics \( D_4 \) and \( D_8 \) produce respectively the unwanted effect of over-computed and under-computed. Furthermore, \( D_8(p_0, p_1) = D_8(p_0, p_2) \) is also very unreasonable. Therefore, for producing a proper MA, the \( D_E \) is suggested\textsuperscript{8} and adopted in this study.

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(a)

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(b)

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(c)

Fig. 1. D-map illustration obtained by (a) \( D_4 \), (b) \( D_8 \), and (c) \( D_E \) metrics.
Based on the original MAT definition, some researchers had explored the theory of skeleton and representation for continuous objects and concluded three significant properties as follows.

1. The connectivity of the derived skeleton is preserved as long as the original object is connected.
2. The original object can be exactly reconstructed from its skeleton.
3. The geometry of the skeleton is invariant under picture rotation.

However because the original idea is derived for the continuous plane, when the digital image is considered it becomes a challenging problem to preserve the above mentioned properties. For example, the distance metric should be used for digital image as discussed previously. Even the proper metric $D_E$ is used, based on the local maxima metric information the obtained MA is no longer to be connected as illustrated in Fig. 2(a). Thus, in this study, an effective ridge-path linking strategy is developed to overcome the disconnectedness problem and its details will be presented in the following subsections.

![Fig. 2. (a) MA result by $D_E$ metric. (b) Six 8-connected groups.](image-url)
2.1. Grouping

According to the definition by Rosenfeld, for a digital image let $P = \{(x_1,y_1),..., (x_t,y_t)\}$ be any $t$-tuple of $(x,y)$s where $t \geq 1$. $P$ is called a 4-path (or 8-path) if for each $r$, where $1 \leq r < t$, we have $|x_r - x_{r+1}| + |y_r - y_{r+1}| \leq 1$ (or $\max(|x_r - x_{r+1}|, |y_r - y_{r+1}|)$), respectively. The elements $p$ and $q$ of pattern $S$ are called 4-connected (or 8-connected) in $S$ if there exists a 4-path (or 8-path) between $p$ and $q$. Based on the 8-connected definition, the connected objects in the MA can be further divided into several groups. For example, consider the MA result in Fig. 2(a), we have six groups, $G_i, i = 1,...,6$, as shown in Fig. 2(b), where each group is 8-connected.

2.2. Ridge path linking

According to the MA property, the $D_E$ of each MA pixel has a local maximum within its neighborhood. Since each connected group is composed of several MA pixels, there exists a shortest ridge path linking between two adjacent groups. Consider two groups, $G_i$ and $G_j$, a ridge path ($RP$) from $G_i$ to $G_j$ is obtained by consecutively searching the next pattern pixel whose $d$-value is the local maximum within the neighbors who do not belong to any group or background pixels. Since multiple ridge paths may exist between the group $G_i$ and any other group $G_j$ (the destination group may be of different), a minimum cost-function is necessarily developed for determining the shortest path. If there are $M$ 8-connected neighbors surrounding the group $G_i$, there are $M$ ridge paths, denoted by $RP_{m}^i$, $m = 1,2,...,M$. Let $N_{m}^i$ be the number of pixels belonging to $RP_{m}^i$. Along the tracing direction of $RP_{m}^i$ from $G_i$ to a $G_j$, let $p_1$ be the first pixel connecting to $G_i$ and $p_{N_{m}^i}$ the last pixel connecting to $G_j$. The cost function called $SD_{m}^i$ (sum of distances between two adjacent pixels in this path) is then defined as follows.

$$SD_{m}^i = \sum_{k=1}^{N_{m}^i-1} D_E(p_k,p_{k+1}) + D_E(p_{N_{m}^i},G_j)$$

Here $D_E(p_{N_{m}^i},G_j)$ represents the minimum Euclidean distance between the last pixel $p_{N_{m}^i}$ and $G_j$. If $SD_{m}^i$ has the minimum value, it represents that the shortest ridge path $RP_{m}^i$ from $G_i$ to $G_j$ will serve as their linking path. Based on the definitions introduced above, the procedure of determining the shortest ridge path linking from $G_i$ to $G_j$ is summarized by the following procedure (abbreviated by RPL).

**RPL: Procedure of finding the link path from $G_i$ to $G_j$**

1. For group $G_i$, if there exist $M$ neighbors surrounding $G_i$, starting from one of these neighbors it can find a ridge path to a $G_j$ along consecutively searching the

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*aWe can imagine that a mountain chain ridges from one mountain to the next one.*
next pattern pixel \( p \) whose \( d \)-value is the local maximum within \( p \)'s neighbors who do not belong to any group or background pixels. There are \( M \) RPs from \( G_i \) to any other group.

2. The pattern pixel having \( d = 1.0 \) or been visited before can not be included in a RP.

3. For pixels in a RP, the last pixel can not be the first pixel. That is, a closed-loop RP is not allowed.

4. Compute \( SD_{im} \) for all \( RP_{im} \), \( m = 1, 2, ..., M \). If \( SD_{im} = \min \{ SD_{im}, \forall m \} \), it indicates that the \( m^* \)-th RP, i.e. \( RP_{im^*} \), is the optimum ridge path between \( G_i \) and the linked group \( G_j \).

Consider the \( 9 \times 9 \) partial D-map including \( G_1 \) and \( G_2 \) from Fig. 2(b), the detail is given in Fig. 3 for further illustration. Here \( G_1 \) has two pattern pixels \((4, 3)\) and \((3, 4)\), whereas \( G_2 \) has one pattern pixel \((2, 6)\). Let \( G_1 \) be the starting group for ridge path tracing, we have 12 possible RPs based on the neighbors surrounding \( G_1 \), that is, \((3, 5), (2, 5), (4, 2), (5, 2), (3, 3), (4, 4), (2, 4), (5, 3), (3, 2), (4, 5), (2, 3)\) and \((5, 4)\), according to the descending order of their \( d \) values. Based on the ridge path defined, we have the following 12 RPs, which are also illustrated respectively as shown in Fig. 4. Note here that in this example, the \( G_2 \) is the only one destination group.

Fig. 3. 12 8-connected neighbors surrounding the group \( G_1 \). The \( d \)-value of \((3, 5)\) nearest to the boundary is 9.85 based on the \( DE \) computation for instance.
Table 1. $SD_{1m}$ computations for all $RP_{1m}$, $m = 1, 2, ..., 12$ from $G_1$ to $G_2$ for the current illustrations given in Fig. 4.

<table>
<thead>
<tr>
<th>$m$</th>
<th>$N_{1m}$</th>
<th>$SD_{1m}$</th>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1.414</td>
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<tr>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>10.828</td>
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<tr>
<td>4</td>
<td>9</td>
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<tr>
<td>5</td>
<td>12</td>
<td>13.656</td>
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<td>4</td>
<td>4.828</td>
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<tr>
<td>7</td>
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<td>11</td>
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<td>12</td>
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$^aRP_{12}$ is the shortest ridge path in this example.

Table 1 lists all the computations of $SD_{1m}$, $m = 1, 2, ..., 12$. In this example, The minimum value of $SD_{12}$ indicates the optimal ridge path linking from $G_1$ to $G_2$ is $RP_{12}$. After all groups are performed by the RPL process, the connected skeleton is thus obtained. However, it is possibly to yield a closed loop between $G_i$ and $G_j$ if a different path linking from $G_j$ to $G_i$ exists for example. In our study, such a situation usually appears at the “fork” or “junction” of a line pattern. Since the yielded hole is very small, it is easily solved by means of filling process to remove
Fig. 4. 12 derived \( RP_s \) for the partial D-map given in Fig. 3.
Fig. 4. (Continued.)
the unwanted hole. After performing the RPL process for all groups in Fig. 2(b), its connected skeleton as shown in Fig. 5 is obtained.

![Connected skeleton obtained by the proposed method.](image)

Fig. 5. Connected skeleton obtained by the proposed method.

![Connected skeleton of pattern “B”, here small dots represent the found linking paths by our method.](image)

Fig. 6. (a) Connected skeleton of pattern “B”, here small dots represent the found linking paths by our method. (b) Thinned result obtained by applying a set of eroding rules given in Table 2.
2.3. Refining

Though the presented ridge path linking method can produce the connected skeleton, it is not guaranteed to be 1-pixel width as the result of pattern “B” given in Fig. 6(a). Here the small dots are used to represent the found linking paths by our method. In order to obtain 1-pixel wide thin line and maintain its connectedness, an existing thinning algorithm\(^5\) can be applied to the skeleton for obtaining a 1-pixel wide thin line. After this refining process, a thinned result as shown in Fig. 6(b) is obtained.

Since the minor branch may be induced near to the fork point in a thinned line, we use a simple distance comparison to further prune it. In this study, let \( f \) be a fork point and \( e \) be the end point of the minor branch from \( f \) to \( e \). If \( D_E(e, f) \leq d(f) + \epsilon \), then the minor branch is pruned. Here \( d(f) \) denotes the \( d \)-value of point \( f \) in the D-map, and \( \epsilon = 3 \) is used in our experiments. As a short summary, based on the Euclidean distance MAT, the proposed approach is composed of generation of D-map, grouping, ridge path linking, and refining as the flowchart given in Fig. 7 to obtain the thinned results of digital patterns.

3. Results and Analyses

The proposed algorithm is implemented by Microsoft Visual Studio 6.0 C++ and run on a laptop computer with Intel\(^\text{®} \) Core\(^\text{TM} \) i5 2.6 GHz CPU and 4G RAM. Since the MAT provides the “medial axis” or “center line” profit for a stroke-based or line-based pattern, it is intuitive that the biased-effect of extracted skeleton using MAT should be smaller than that obtained by a rule-based (or mask-operated) thinning algorithm as Chen and Hsu investigated.\(^8,10\) Therefore, a similar skele-
tonization work using distance map presented by Chang\textsuperscript{4} is first used for comparison. Chang’s work defined four patterns of prominent sign barriers and checked them on a scanline, where ridge points will be extracted from the distance map by detecting prominent sign changes on two orthogonal scanlines and connected them based on the topology of distance map. Then the graph is converted to a set of snakes and deformed on the distance map to represent the skeleton. We take one test image from Chang’s work and perform it by our method and Chang’s method.\textsuperscript{4} The thinned results are shown in Fig. 8(a) and 8(b), respectively. By examining the details of the results, we find that both methods can obtain a connected thin skeleton but some branches appear in the Chang’s result. In addition, a rule-based thinning\textsuperscript{27} modified from the rotation invariant rule-based thinning algorithm,\textsuperscript{1} is also used for further comparison. This thinning method is originally presented to preserve the topology of symbols and letters written in any language. Fig. 9(a) and 9(b) show the resultant thinned patterns “Chen” (a Chinese character) obtained by the proposed method and the rule-based method.\textsuperscript{27} respectively. Here we also find that several branches of thinned result appear in the rule-based method. From the above two comparisons, it is therefore worthy of further investigating the merit of reducing the branching effect by the proposed algorithm. Since the proposed approach is considered to improve the thinning effect, in the following the rule-based thinning\textsuperscript{27} is used for assistance to investigate the inherent properties of our method, which are branching effect and thus the related boundary noise immunity.

Fig. 8. Thinned result obtained by (a) the proposed method, and (b) Chang’s skeletonizing method.

3.1. Branching effect

To investigate the branching effect of a thinned line, we can make a set of original thin lines and thicken them to produce a set of thickened patterns for analysis.
as suggested by Chen’s work.\textsuperscript{10} By Chen’s thickening procedure, we designate 16 thin line patterns having angles ranged within 15°, 20°, \ldots, 90°, and produce the corresponding thicken patterns as shown in Fig. 10(a). Thinned results obtained by the proposed method and the rule-based method\textsuperscript{27} are displayed in Fig. 10(b) and 10(c), respectively. We inspect the details of thinned results in Fig. 10(b) and 10(c), we find that there are 13 branches happening at the patterns from 15° to 75° by rule-based method, whereas only 5 branches happen within [15°, 35°] by our method. Such a phenomenon can be explained that the presented approach adopts a more global information to determine the medial curve, whereas a local operation, such as 3 × 3 or 4 × 4 mask operation, used in a rule-based thinning dominates the thinning result. The latter exists an inherent issue of balance speed of deletions operated by a thinning algorithm as argued by Chen.\textsuperscript{10} Accordingly, the result confirms that the branching effect can be reduced by our method.

3.2. \textit{Boundary noise immunity}

Noise immunity is sometimes discussed in the literatures.\textsuperscript{11,14,26,10} Due to the small mask is operated in a thinning algorithm, the usually concerned noise is the one-pixel boundary noise. A short conclusion on the noise immunity made by Poty and Ubeda\textsuperscript{26} is as follows. The up-to-date thinning operators cannot differentiate an end pixel of a skeleton from a noisy pixel of the boundary. Such a problem cannot be avoided by using neighborhood at unit distance except with a smoothing process on the original boundary before thinning. In our early study on thinning noisy digital pattern,\textsuperscript{9} which is based on the principles of human visual perception,\textsuperscript{23} and is achieved fundamentally by means of (1) finding the effective circular range containing maximal pixel information; (2) computing the symmetry information within
Fig. 10. (a) Thickened digital patterns with a variety of angled lines. Results obtained by (b) the proposed method, and (c) the rule-based thinning method.
the found range; and (3) performing a gray-scale thinning for post-processing. In this study, the use of MA information should provide a better noise immunity since it is conveyed a more global information like the concept adopted in Chen’s early study. In the following we present an experiment for the investigation of boundary noise immunity.

Consider a clear binary digital pattern as shown in Fig. 11(a), a specified degree of noise (denoted by signal to noise ratio, or $SNR$) is incorporated into the boundary to form a pattern having boundary noise. The noise added to the digital pattern are generated by a random generator in C Language. $SNR$ is defined as follows.

$$SNR = 10 \log \frac{Signal}{Noise}$$  \hspace{1cm} (2)

Here $Signal$ and $Noise$ represent the number of original boundary pixels and boundary pixels being changed (noise added), respectively. Fig. 11(b) and 11(c) show re-
spectively the thinning results by the proposed approach and the rule-based method for the boundary noise added patterns with different SNRs. In this experiment, it is apparent that not only the rule-based thinning yields a more bias effect but also produces more unnecessary small branches. This experiment confirms that the proposed approach can obtain a more reasonable thinning result.

In order to further statistically measure the performance of our method on boundary noise immunity, a measurement of skeleton deviation (MSD) is defined. Given a pattern $P$, let $Q$ denote its boundary noise embedded version. In addition, let $S_P$ and $S_Q$ be the skeleton of $P$ and that of $Q$. Then the MSD can be computed as follows.

$$MSD(S_Q, S_P) = \sum_{q \in S_Q} D_E(q, S_P)$$

(3)

where $D_E(q, S_P)$ represents the minimum Euclidean distance between $q$ and $S_P$. For a thinning algorithm, the lower MSD means that it is more better to possess the ability of boundary noise immunity since the skeleton variation from the original pattern to the boundary noise embedded one is smaller. Based on the MSD, some patterns come from Chars74K OCR dataset\textsuperscript{12} (refer to Fig. 12) and MPEG7 CE-Shape-1 dataset\textsuperscript{20} (refer to Fig. 13) as well as their 10dB boundary noise embedded versions are adopted for the measurements and comparisons. Note here that for an original pattern it is inevitable that some boundary points will affect the thinning result if the boundary smoothing process is not applied before. Four algorithms, namely Chen and Hsu\textsuperscript{7}, Jang and Chin\textsuperscript{15}, Rockett\textsuperscript{27}, and the proposed method are used for the experiments. The thinning results are shown in Fig. 12 and Fig. 13. The corresponding MSD measurements are reported in Table 2 and Table 3, respectively. It can be observed that the proposed method possesses a better boundary noise immunity to obtain a qualified thinning result (with less branching effect). In addition, the time of executing the algorithm on computer are also reported in Table 4 and Table 5. It can be seen that the execution time of our method is longer than that of others. It is reasonable since the complexity of our algorithm is indeed higher than that of rule-based thinning algorithms. As a result, a good boundary noise immunity for thinning digital patterns can be obtained by our dedicated MAT-based thinning algorithm.

4. Conclusions and Future Works

To overcome the disconnected issue of MAT-based skeletonization, an approach including generation of D-map, grouping, ridge-path linking, and refining has been presented; and our experiments confirm that a 1-pixel wide thin line can be effectively obtained for line patterns. However, two another significant issues are still remained to be resolved. First, how the trend of thinning result “matches with” that of the original line pattern. Even some thinning approaches claimed possess the rotation invariant characteristics, however from the viewpoint of shape the thinning
Fig. 12. Thinning results of some patterns from the Chars74K OCR dataset obtained by means of (a) Chen and Hsu, (b) Jang and Chin, (c) Rockett, and (d) the proposed method. Here the original patterns and the corresponding 10dB noise-embedded ones are placed at the first and second row, respectively.
Fig. 13. Thinning results of some patterns from the MPEG7 CE-Shape-1 dataset obtained by means of (a) Chen and Hsu, (b) Jang and Chin, (c) Rockett, and (d) the proposed method. Here the original patterns and the corresponding 10dB noise-embedded ones are placed at the first and second row, respectively.
Table 2. MSD measurements for the thinned results given in Fig. 12.

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Chen and Hsu(^7)</th>
<th>Jang and Chin(^{15})</th>
<th>Rockett(^{27})</th>
<th>Proposed Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>“1”</td>
<td>186.4</td>
<td>2020.9</td>
<td>533.7</td>
<td>65.0</td>
</tr>
<tr>
<td>“3”</td>
<td>166.1</td>
<td>1091.0</td>
<td>274.8</td>
<td>76.0</td>
</tr>
<tr>
<td>“5”</td>
<td>170.7</td>
<td>1250.5</td>
<td>435.4</td>
<td>75.0</td>
</tr>
<tr>
<td>“A”</td>
<td>154.3</td>
<td>1933.7</td>
<td>288.3</td>
<td>76.0</td>
</tr>
<tr>
<td>“B”</td>
<td>197.5</td>
<td>1750.8</td>
<td>399.3</td>
<td>85.0</td>
</tr>
<tr>
<td>“K”</td>
<td>251.5</td>
<td>1430.1</td>
<td>577.3</td>
<td>94.0</td>
</tr>
<tr>
<td>“n”</td>
<td>381.6</td>
<td>3616.5</td>
<td>1070.0</td>
<td>58.0</td>
</tr>
<tr>
<td>Average</td>
<td>215.4</td>
<td>1870.5</td>
<td>511.3</td>
<td>75.6</td>
</tr>
</tbody>
</table>

Table 3. MSD measurements for the thinned results given in Fig. 13. Here the “pattern number” is ordered from the left to right for the patterns in Fig. 13.

<table>
<thead>
<tr>
<th>Pattern number</th>
<th>Chen and Hsu(^7)</th>
<th>Jang and Chin(^{15})</th>
<th>Rockett(^{27})</th>
<th>Proposed Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4940.8</td>
<td>8099.1</td>
<td>4990.3</td>
<td>62.4</td>
</tr>
<tr>
<td>2</td>
<td>935.7</td>
<td>7014.3</td>
<td>937.0</td>
<td>114.0</td>
</tr>
<tr>
<td>3</td>
<td>808.5</td>
<td>2491.0</td>
<td>1525.3</td>
<td>196.3</td>
</tr>
<tr>
<td>4</td>
<td>787.3</td>
<td>15408.6</td>
<td>3906.5</td>
<td>237.9</td>
</tr>
<tr>
<td>5</td>
<td>6704.8</td>
<td>6475.9</td>
<td>9249.8</td>
<td>196.1</td>
</tr>
<tr>
<td>6</td>
<td>4236.4</td>
<td>7295.0</td>
<td>4646.0</td>
<td>259.0</td>
</tr>
<tr>
<td>7</td>
<td>316.2</td>
<td>4960.7</td>
<td>624.5</td>
<td>193.4</td>
</tr>
<tr>
<td>Average</td>
<td>2675.7</td>
<td>7392.1</td>
<td>3697.1</td>
<td>179.9</td>
</tr>
</tbody>
</table>

Table 4. Execution time (sec) for the thinned results given in Fig. 12.

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Chen and Hsu(^7)</th>
<th>Jang and Chin(^{15})</th>
<th>Rockett(^{27})</th>
<th>Proposed Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>“1”</td>
<td>0.145</td>
<td>0.102</td>
<td>0.113</td>
<td>0.487</td>
</tr>
<tr>
<td>“3”</td>
<td>0.104</td>
<td>0.074</td>
<td>0.067</td>
<td>0.274</td>
</tr>
<tr>
<td>“5”</td>
<td>0.119</td>
<td>0.080</td>
<td>0.068</td>
<td>0.329</td>
</tr>
<tr>
<td>“A”</td>
<td>0.124</td>
<td>0.093</td>
<td>0.080</td>
<td>0.374</td>
</tr>
<tr>
<td>“B”</td>
<td>0.132</td>
<td>0.087</td>
<td>0.084</td>
<td>0.409</td>
</tr>
<tr>
<td>“K”</td>
<td>0.109</td>
<td>0.077</td>
<td>0.065</td>
<td>0.313</td>
</tr>
<tr>
<td>“n”</td>
<td>0.143</td>
<td>0.100</td>
<td>0.097</td>
<td>0.583</td>
</tr>
<tr>
<td>Average</td>
<td>0.125</td>
<td>0.088</td>
<td>0.082</td>
<td>0.396</td>
</tr>
</tbody>
</table>

results for different patterns should be rather different but they look like similar ones. It is thus worthy of further studying especially for the multi-font characters. Following this issue, it conducts another question that “is it possible that the original line pattern can be completely restored with the thinned result?” It is not easy for a traditional rule-based thinning algorithm since not any restorable information is preserved but the connectivity property during the thinning process. Accordingly how to develop a restorable and trend-preserved skeletonization algorithm for line
Table 5. Execution time (sec) for the thinned results given in Fig. 13. Here the “pattern number” is ordered from the left to right for the patterns in Fig. 13.

<table>
<thead>
<tr>
<th>Pattern number</th>
<th>Chen and Hsu$^7$</th>
<th>Jang and Chin$^{15}$</th>
<th>Rockett$^{27}$</th>
<th>Proposed Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.298</td>
<td>0.170</td>
<td>0.264</td>
<td>1.908</td>
</tr>
<tr>
<td>2</td>
<td>0.270</td>
<td>0.145</td>
<td>0.191</td>
<td>1.079</td>
</tr>
<tr>
<td>3</td>
<td>0.274</td>
<td>0.197</td>
<td>0.176</td>
<td>0.964</td>
</tr>
<tr>
<td>4</td>
<td>0.465</td>
<td>0.236</td>
<td>0.360</td>
<td>3.250</td>
</tr>
<tr>
<td>5</td>
<td>0.381</td>
<td>0.198</td>
<td>0.293</td>
<td>1.624</td>
</tr>
<tr>
<td>6</td>
<td>0.429</td>
<td>0.270</td>
<td>0.350</td>
<td>1.697</td>
</tr>
<tr>
<td>7</td>
<td>0.349</td>
<td>0.168</td>
<td>0.213</td>
<td>1.121</td>
</tr>
<tr>
<td>Average</td>
<td>0.352</td>
<td>0.198</td>
<td>0.264</td>
<td>1.663</td>
</tr>
</tbody>
</table>

patterns could be a good topic in the thinning field and be regarded as our future works.

References

34. C.Y. Suen and P.S.P. Wang, *Thinning Methodologies for Pattern Recognition*, Se-
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